International Journal of Emerging Technology in Computer Science & Electronics (IJETCSE)
ISSN: 0976-1353 Volume 8 Issue 1 –APRIL 2014.

MATHEMATICAL MODELLING OF BALL ON A MIDDLE SUPPORTED BEAM

Chinju Joseph[#1], Nitha Thomas[*2] ,Jefin Thomas[#3],ElizabathRajan[#4] , ReemaMathew.A [*5]

[#1]*P.G scholar, Electronics & Instrumentation, VimalJyothiEngg. College Chemperi, Kannur,India*

[*2]*P.G scholar, Electronics & Instrumentation, VimalJyothiEngg. College Chemperi, Kannur,India*

[#3]*P.G scholar, Electronics & Instrumentation, VimalJyothiEngg. College Chemperi, Kannur,India*

[*4]*P.G scholar,Electronics& Instrumentation, VimalJyothiEngg. College Chemperi, Kannur,India*

[*5]*HOD,Electronics& Instrumentation, VimalJyothiEngg. College Chemperi, Kannur,India*

***Abstract*— The ball and beam system is one of the most enduringly popular and important laboratory models for teaching control systems engineering. It is widely used because it is very simple to understand, and the control techniques that can be studied it cover many important classical and modern design methods. This system has a very important property; it is open loop unstable. It is very simple system that is, a steel ball rolling on the top of a long beam. The beam is mounted on the shaft of an electrical motor the beam can be tilted about its Centre axis by applying an electrical control signal to the motor amplifier. This paper describes how to obtain the equations of motion for the Ball and Beam process. There are two different methods for obtaining equations of the model. One of them is derived with simple mathematic equations using Newton's second law and another through the use of the Lagrangian Method.**

***Index Terms*—ball and beam, Lagrangian method, Mathematical model, Newton's law,**

I. INTRODUCTION

The ball and beam system is also called 'balancing a ball on a beam'. It can usually be found in most university control labs. This system is generally linked to real control problems such as horizontally stabilizing an airplane during landing and in turbulent airflow[1]. The system is often used as a bench mark problem for many different control schemes such as the control model in the rocket toppling control system,[2] where a feedback system is used to prevent rockets to topple out of balance during launch by forces and moments that could perturb the vertical motion.

There are two degrees of freedom in this system.

1) Ball rolling up and down the beam,
2) Beam rotating through its central axis.

The main aim of the system is to control the position of the ball to a desired reference point, and reject external disturbances such as a push from a finger.

The control signal can be derived by feeding back the position information of the ball. The control voltage signal given to the DC motor via a power amplifier, the torque generated from the motor drives the beam.it will rotate the beam to the desired angle. Thus, the ball can be located at the desired position. It is important to point out that the open loop of the system is unstable and nonlinear.

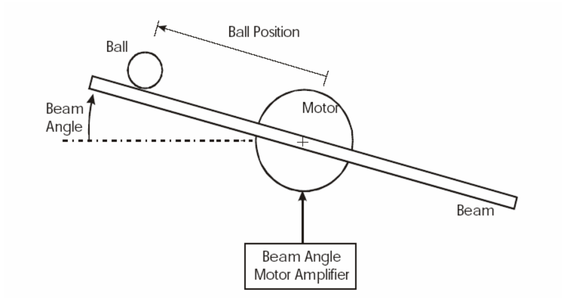


Fig.1:diagram of ball and beam

The fig.1 shows a typical ball and beam diagram. Here ball is balancing at the center of a beam. Here beam is supporting at the middle [2]. Beam is making some angle with horizontal axis. At a particular angle, ball gets balanced.

There are two configurations to support the beam. One configuration is shown in the Fig. 2, which illustrates that the beam is supported in the middle, and rotates against its central axis. Most ball and beam systems use this type of configuration. The advantage of this form is that it is easy to build, and the mathematical model is relatively simple.

The other configuration is shown in Fig.3. The beam is supported on the both sides by two level arms.[5] One of level arms is pined, and the other is coupled to output gear. The disadvantage of this configuration is that more consideration of the mechanical parts is required, and this may add some difficulties in deriving a mathematical model. The advantage of this system is that relatively small motor can be used due to the leverage effect.

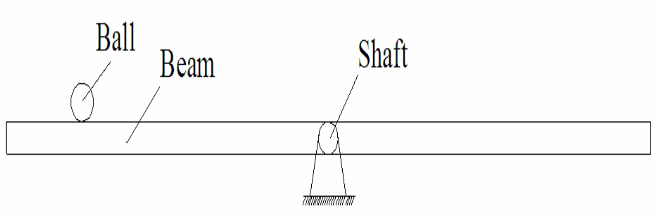


Fig2:beam supporting at the middle

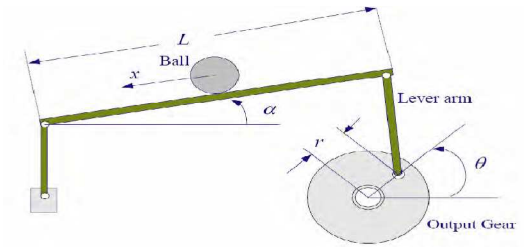


Fig3: beam supported on both sides

Based on the shaft supporting the beam in the middle, there are two further configurations. One is, position the beam in the middle of two supports. The advantage of this system is that the beam is supported does not experience a moment [6]. The second configuration cantilevers the beam off the shaft. This configuration has been employed in this project because of the simplified mechanical design.

In this ball and beam system we use Stainless steel ball of diameter 20mm which is placed on the Polycarbonate beam in the Aluminum channel has the length of 700mm. Aluminum channel, is made of the standard Aluminum channel [7]. A DC motor with the gearbox and the digital encoder is commercial available for tilting the beam.

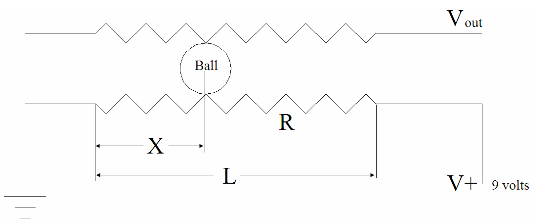


Fig4.linear potentiometer

Position of the ball measured by the help of resistive wires placed on the beam. Resistive wires made of nickel-chromium Position sensor amplifier, is used to amplify and filter the high frequenciesof the ball's position signal [2]. Two types of sensors are involved in this application, the position sensor which measures the position of the ball on the beam and the angle sensor which measures the rotational position of the beam. Cheap, reliable, and good resolution sensors are required in this application.

In the next chapter, a theoretical analysis of the ball and beam system will be presented. The mathematical model of the system is derived to model the system. The model will be further simplified to achieve a better controllability.

II. MATHEMATICAL MODELLING

The ball and beam system should be fully understood before attempting to control it. A theoretical analysis is the first step to approach this 'black box' system.Usually, analytical processes require engineers to investigate a system based on universal laws of physics and their own experience. it is essential to build a mathematical model of the system to express the relationships between all components. Usually, there are several techniques used to derive the mathematical model[3]: the transfer function between input and output. The simplest way to deliver the mathematical model is to employ physics and electronic laws to express the system. In this case, the system is very simple, thus this method of delivering is most efficient to derive a mathematic model. Some other methods, such as a system identification method[10], [11] or experimental method, are applied in more complex systems, which are impossible to derive an accurate model by simple laws. It is worthwhile to note that the model derived in the following is only an ideal model regardless which kind of method is used. In other words, it is impossible to build a perfect model.

A. *Newton's second law*

Here we are considering the relationship between ball position and beam angle[3].

Parameters that taken under consideration are the following:α - Beam angle coordinate, L - Beam Length, m - Mass of the ball, R - Radius of the ball, J - Ball's moment of inertia, G - Gravitational acceleration, x - Position of the ball.

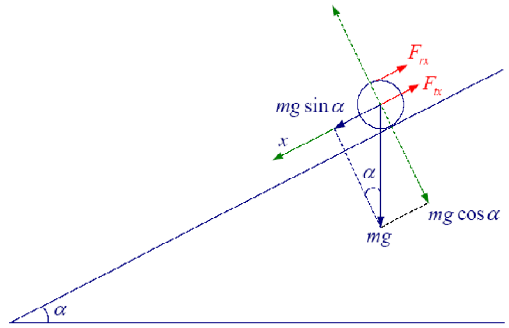


Fig 5.free body diagram of ball and beam system.

Free body diagram of ball and beam system is shown in the fig.5 above. Suppose our ball is in its equilibrium point. Consider all forces acting on system. 'mg' the weight of ball which is acting downwards. mgsin α Neglecting frictional forces, the two forces influencing the motion of the ball are:

- Ftx Force due to translational motion
- Frx Force due to ball rotation.

Force due to translational is:

$$\ddot{x}=\frac{d^2x}{dt^2}$$

$$F_{tx}=\text{m}.\ddot{x} \tag{1}$$

Torque due to rotation of ball is,

$$T_r = F_{rx}.\text{R}=\text{J}.\frac{d_{wb}}{dt} = \frac{J}{R}\ddot{x}$$

So the force due to ball rotation is,

$$F_{rx}=\frac{J}{R^2}\ddot{x} \tag{2}$$

Moment of inertia of the sphere is,

$$\text{J}=\frac{2}{5}mR^2 \tag{3}$$

By substituting (3) in (2)

It gives,

$$F_{rx}=\frac{2}{5}m\ddot{x}$$

By Newton's second law force balance along inclination is,

$$F_{rx}+F_{tx}=m.g\ \sin\alpha(4)$$

Substituting the above equations in (4) we get,

$$\ddot{x}=\frac{5}{7}g.\sin\alpha \qquad (5)$$

The above (5) is derived from the relation of ball position and beam angle. Next step is to derive the relation between beam angle with motor voltage. So the total transfer function is the product of model of the ball position with respect to beam angle and the model of the angle process with respect to the motor voltage, ie,

$$H(s)=H_{\varphi}(s)\times H_x(s) \qquad (6)$$

Where,
$H_{\varphi}(s)$ is the Model of the angle process with respect to the motor voltage
$H_x(s)$ is the Model of the ball position with respect to the beam angle

Model of beam angle vs. input voltage

The relationship between the input voltage and the angle of the beam is defined by the DC motor transfer function[13]. The DC motor, used for angle control application may be thought of as the 'dirty integrator' or the integrator with a filter action as shown in fig 6.

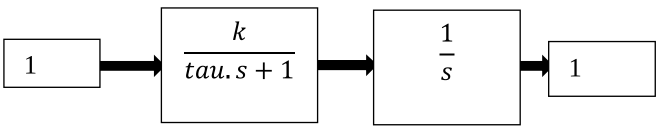


Fig.6. general DC motor block diagram for angle control

After substituting the measured parameters on the actual model we get the transfer function of the DC motor as,

$$H_{\varphi}(s)=\frac{\theta(s)}{V(s)}=\frac{11.54}{s(1+0.4s)} \qquad (7)$$

Also from (5) taking Laplace transform we get,

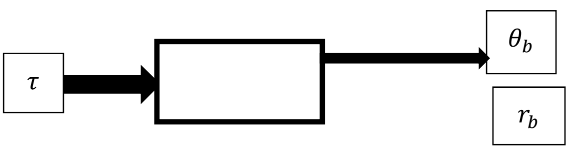
$$H_x(s)=\frac{x(s)}{\theta(s)}=\frac{5/7.g}{s^2} \qquad (8)$$

Finally we get overall transfer function of the system,

$$H(s)=\frac{10.5}{s^3(0.4s+1)} \qquad (9)$$

B. *Lagrangian methods*

The control objective is to control the torque τ applied at the pivot of the beam, [4]such that the ball can roll on the beam and track a desired trajectory. The torque causes thus a change of the beam angle and a movement in the position of the ball.



The parameters of the ball and beam are defined as follows: Ib - Beam's moment of inertia, θb - Beam angle, ms - Mass of the ball, Is - Ball's moment of inertia, rs - Radius of the sphere, rb - Position of the ball.

Kinetic energy of the system is,

$$T_{total}=T_{beam}+T_{sphear} \qquad (10)$$
$$T_{beam}=T_{(beam)frame}+T_{(beam)body\_center}$$
$$T_{sphear}=T_{(sphear)frame}+T_{(sphear)body\_center}$$

Each energy is,

$$T_{(beam)frame}=0$$
$$T_{(beam)body\_center}=\frac{1}{2}.I_b.\dot{\theta_b}^2$$
$$T_{spere,frame}=\frac{1}{2}.(m_s.{r_b}^2).\dot{\theta_b}^2+\frac{1}{2}.m_s.\dot{r_b}^2$$
$$T_{(sphear)body\_center}=\frac{1}{2}.I_s.\dot{\theta_s}^2 \qquad (11)$$

Where,

$$I_s=\frac{2}{5}.(m_s.{r_b}^2)$$
$$\dot{r_b}=r_s.\dot{\theta_s}$$

Then (11) become,

$$T_{(sphear)body\_center}=\frac{1}{2}\left[\frac{2}{5}.(m_s.\dot{r_b}^2)\right]$$

So total kinetic energy of the system is,

$$T_{total}=\frac{1}{2}[(I_b+m_s.{r_b}^2).\ \dot{\theta_b}^2+\frac{7}{5}.m_s.\dot{r_b}^2]$$

The rolling ball alone exhibits the potential energy of the system:

$$V=m_s.\ g.r_b\sin\theta_b$$

To write the equations of motion, we define the Lagrangian L, to be the difference between the kinetic and potential energy of the system.

$$L(q,\dot{q})=T(q,\dot{q})\text{-}V(q)$$

where q denote the so-called generalized coordinates of the system, T is the kinetic energy of the system and V is the potential energy of the system.
by substituting equations of energy, we get,

$$L=\frac{1}{2}[(I_b+m_s.{r_b}^2).\ \dot{\theta_b}^2+\frac{7}{5}.m_s.\dot{r_b}^2]-m_s.\ g.r_b\sin\theta_b \qquad (12)$$

Lagrange's equations of motion are formed from:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{q_i}}\right)-\frac{\partial L}{\partial q_i}=F_{qi}$$

where F_{qi} is the external force, in this case F_{qi} is τ.
qi in this case are θ_b and r_b.

From these equations we get equation of motion as:

$$(I_b+m_s.{r_b}^2).\ \ddot{\theta_b}+2.m_s r_b\dot{r_b}\dot{\theta_b}\text{-}m_s.\ g.r_b\cos\theta_b=\tau \qquad (13)$$
$$\ddot{r_b}+\frac{5}{7}(g.\ \sin\theta_b-r_b\dot{\theta_b}^2)=0 \qquad (14)$$

Nonlinear states are $\theta_b,\dot{\theta_b}$,r , $\dot{r}$

Then the state equations become,

$$\begin{cases} \dfrac{d\theta}{dt} = \dot{\theta} = f_1 \\ \dfrac{d\dot{\theta}}{dt} = \ddot{\theta} = \left(\dfrac{2m_s.r_b}{I_b + m_s.r_b^2}\right).\dot{\theta}\dot{r}_b - \left(\dfrac{m_s.g.r_b.\cos\theta_b}{I_b + m_s.r_b^2}\right) + \left(\dfrac{1}{I_b + m_s.r_b^2}\right).\tau = f_2 \\ \dfrac{dr}{dt} = \dot{r} = f_3 \\ \dfrac{d\dot{r}}{dt} = \ddot{r} = \dfrac{5}{7} r_b \dot{\theta}_b^{\,2} - \dfrac{5}{7}.g.\sin\theta_b = f_4 \end{cases}$$

To have a desired equilibrium in:

$\theta_e = 0$ $\quad\quad$ $\dot{\theta}_e = 0$

$r_e = 1$ $\quad\quad$ $\dot{r}_e = 0$

and

$$x_1 = \theta - \theta_e = \theta$$
$$x_2 = \dot{\theta} - \dot{\theta}_e = \dot{\theta}$$
$$x_3 = r - r_e = r - 1$$
$$x_4 = \dot{r} - \dot{r}_e = \dot{r}$$

Thus we get linearized state space model as,

$$\begin{bmatrix} \dot{\theta}_b \\ \ddot{\theta}_b \\ \dot{r} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \dfrac{-m_s.g}{I_b + m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{-5}{7}.g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_b \\ \dot{\theta}_b \\ r \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{I_b + m_s} \\ 0 \\ 0 \end{bmatrix} \tau \quad (15)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_b \\ \dot{\theta}_b \\ r \\ \dot{r} \end{bmatrix}$$

III. RESULT

Roots of this system are showing in the fig.7.It is clear that among the four roots two of the poles are locating on margin and one on the left side and other on the right side.

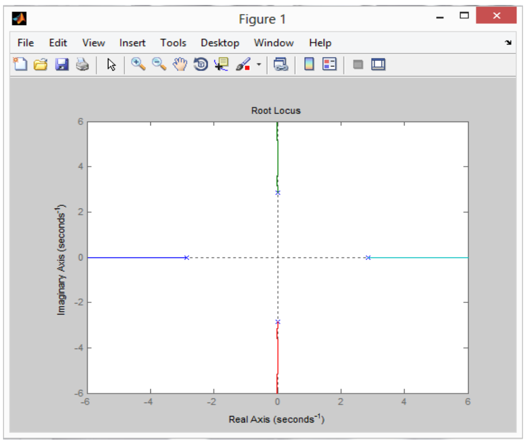


Fig 7. Poles of the system.

Fig 8 shows the step response of the system.First graph in fig8 is the angle of the beam changing with respect to input.it is clear that angle value goes to infinity since there is no control action. Similarly the second graph shows position change of the ball. Uncontrollability of position of ball is clear from the graph.

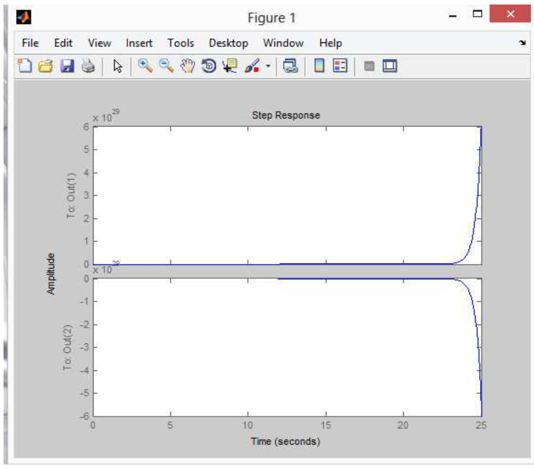


Fig.8:step response of the system.

IV CONCLUSION

Before go for the control of system its modelling is that much important The main goal of this paper was to do the modeling of the Ball and Beam processand to identify the various control strategies applicable to control this nonlinear open loop unstable system. Objective has been fulfilled by the help of newton's laws of motion and the Lagrangian method. Figures 7 and 8 gives us the clearcut nonlinear nature of our system. So, these results shows that better control action is needed to control this nonlinear system.

V REFERENCES

[1] peter wellstead Ball and beam 1:basics,control system principles,co.uk.

[2]Master thesis project:Wei Wang ,School of Mechanical Engineering, The University of Adelaide ,June- 2007.

[3]Marta Virseda,Modeling and Control of theBall and Beam Process, Department of Automatic Control,Lund Institute of Technology,March 2004

[4]Ambalavanar, S., Moinuddin, H. M. & Malyshev, A. Ball and Beam Balancer, BAThesis, Lakehead University-2006,

[5]Arroyo, S. (2005), *Ball on Balancing Beam*, viewed 12 December 2006,

[6]Master Thesis Project:*Xavier Callier*. Department of Automatic Control. Lund Institute of Technology.August 2002 – January 2003

[7]Master Thesis Project: *Francesco Calugi*. Department of Automatic Control. Lund Institute of Technology. October 2001 – April 2002

[8]A. Isidori, Nonlinear Control System, Springer-Verlag, New York, 1989.

[9] H. Nijmeijer, A.J. Van Der Schaft, Nonlinear Dynamical Control Systems, Springer-Verlag, New York, 1990.

[10] J.J.E. Slotine, W. Li, Applied Nonlinear Control, Prentice-Hall, New York, 1991.

[11] H.K. Khalil, Nonlinear Systems, Prentice-Hall, New Jersey, 1996.

[12] Hauser, J., Sastry, S., Kokotovic, P., 2002. Nonlinear control via approximate input–output linearization: the ball and beam example. IEEE Transactions on Automatic Control 37 (3), 392–398.

[13]University of Alberta (2006), '*EE357 Control System I: Lab Manual*', viewed 4 December 2006.