

LOW COMPUTATIONAL IMPLEMENTATION OF DIGITAL PWM TO REDUCE DISTORTIONS

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Abstract— Digital Pulse Width Modulator transforms non constant amplitude signals into pulse signals, with the information of non constant amplitude signals lying on the width of the pulsed signals. Additional processing steps are required to reduce the inherent aliasing effect. This brief describes an alternative method for the implementation of a digital aliasing free pulse width modulator which proves to reduce the computational complexity further than multiplier less implementation of digital aliasing free pulse width modulator which uses look up tables, adders and arithmetic shifters only. Here two dimensional look up tables are used instead of one dimensional look up tables to reduce the complexity.

Keywords—Aliasing Free Pulse Width Modulation(PWM),Radio Frequency (RF),Power Amplifiers (PAs).

I. INTRODUCTION

Digital pulse width modulation (PWM) can be used to create the switching mode power amplifiers (PAs), used in, eg. Digital audio applications, or to generate the pulsed signals required in burst mode radio frequency (RF) transmitters. For these applications, a digital pulse width modulator encodes a non constant amplitude input signal into two level pulsed signals by encoding amplitude information into pulses of different widths. The pulsed signals or a pass band equivalent of it is applied to the PA. After the PA, the amplified input signal, or its pass band equivalent, respectively, can be retrieved by using an adequate filter. The high quality of the retrieved signal is crucial for the aforementioned applications.

The phase modulated and RF upconverted pulsed signal can be used to drive an RF PA in burst mode operation. After amplification, the desired signal, which is the amplified pass band equivalent of the original baseband signal has to be recovered by a band pass filter. In the process of creating the pulsed signal, a large amount of additional frequency content is added to the original baseband signal. The filtering operation is feasible only if the distortion around the desired signal is low enough. A measure of amount of distortion is the dynamic range (DR), which is defined as the distance between the average power of the desired signal and the power of the surrounding distortion given in dB. It is therefore of utmost importance that the amplified signal

exhibit's a sufficient good DR to enable satisfying signal recovery quality and to fulfill the tight spectrum emission requirements posed by modern communication standards. This is only possible if the driving supplied to the PA provide excellent spectral characteristics.

There are several well-established methods to encode a non constant amplitude signal into a pulsed signal, such as $\Sigma\Delta$ modulation [9] or various comparator-based PWM methods [5]. All these methods, however, have in common that they inherently suffer from distortion in and around the frequency band of the input signal, which prevents retrieving a high quality signal after amplification and the filtering operation [8], [11]. Hence, a solution that eliminates the distortion while maintaining low computational effort has to be found. Several methods have been proposed to reduce or eliminate the effects of the distortion. In [1], a noise cancellation circuit that requires additional analog hardware is proposed. The authors of [7] present an algorithm to optimize the noise shaped coding performance of $\Sigma\Delta$ modulators. In [9], a $\Sigma\Delta$ modulator comprising noise shaping loop filters in combination with a digital feed forward error correction method is introduced. This concept is further elaborated in [9]. In [8], the authors introduce a modified version of asymmetric double-edge PWM that eliminates all destructive aliasing distortion but requires a large number of multiplications and trigonometric functions, which increases the computational effort compared to a comparator. In [12] the method proposed in [8] is implemented by using lookup tables (LUTs), adders, and arithmetic shifts only. In this brief two dimensional look up tables are used instead of one dimensional look up tables to reduce the computational complexity further. The suggested implementation therefore allows for generating practically distortion-free digital pulse width-modulated signals in a computationally efficient way.

II. PRINCIPLE OF PULSE WIDTH MODULATION

The pulse width modulation encodes a non constant envelope signal into train of two level pulses with varying widths, such that the width of the pulses represents the magnitude of the signal. The input signal to the pulse width modulator $a(t)$ i.e., the modulating signal can be demodulated by the integration over the pulsed signal $y(t)$, which can be achieved by using an adequate filter. Consider asymmetrical pulses that are centered around the midpoint of a fixed interval, the pulse period T_p . The transitions between the two levels 0 and 1,

and vice versa , are called leading edge (LE) and trailing edge (TE) respectively . Both edges are determined by finding the intersection points of the modulating signal $a(t)$ with triangular reference waveform $r(t)$, where $a(t)$ and $r(t)$, where $a(t)$ and $r(t)$ are bounded by $[0,1]$. The higher the amplitudes, the longer the arising pulse widths.

The triangular reference waveform $r(t)$ is given in the equation 1

$$r(t)=\begin{cases} (-2t/Tp) + 2m + 1, mTp \leq t < mTp + (Tp/2) \\ (2t/Tp) - 2m - 1, mTp + (Tp/2) \leq t < (m + 1) Tp \end{cases} \quad (1)$$

Where m denotes the m th interval, such that LE and TE of the m th pulse occur at the time instant which is given in equations 2 and 3

$$tLEm=mTp+(Tp/2)-a(tLEm)Tp/2 \quad (2)$$

$$tTEm=mTp+(Tp/2)+a(tTEm)Tp/2 \quad (3)$$

An operator $P(a(t),r(t)) = 1$, for $a(t) > r(t)$ and 0 , for $a(t) < r(t)$.Such that the pulsed signal $y(t)$ is obtained which is expressed by equation 4

$$y(t)=P(a(t),r(t)) \quad (4)$$

A. Time domain analysis

A detailed analysis of the signal generated by the pulse width modulator requires an analytical description of PWM process. Hence it is demonstrated that a Fourier series can be used to derive an analytical closed form description of the asymmetric double edge process .By introducing two different time variables τ_1 and τ_2 , the PWM operator P can be written as a two dimensional operator where the signal $y(t)$ can be obtained according to the equation 5

$$y(t)=P(a(\tau_1),r(\tau_2))|\tau_1=\tau_2=t \quad (5)$$

Since the reference wave $r(t)$ is periodic with period Tp , it follows that $P(a(\tau_1),r(\tau_2)) = P(a(\tau_1), r(\tau_2+mTp))$. Thus, for any τ_1 , $P(a(\tau_1),r(\tau_2))$ is a periodic function of τ_2 and $P(a(\tau_1),r(\tau_2))$ can be expanded into a Fourier series which is given in equations 6,7 and 8

$$P(a(\tau_1),r(\tau_2)) = \sum_{k=-\infty}^{\infty} ck(a(\tau_1))ej2\Pi k \tau_2 /Tp \quad (6)$$

Where the coefficients $ck(a(\tau_1))$ is given by

$$ck(a(\tau_1))=1/Tp \int_0^{Tp} P(a(\tau_1), r(\tau_2))e^{-j2\Pi k \tau_2 /Tp} d\tau_2 \quad (7)$$

$$ck(a(\tau_1))=(1/\Pi k)\sin(\Pi k a(\tau_1))ej\Pi k \quad (8)$$

By substituting the equation 8 in equation 6, equation 9 is obtained

$$P(a(\tau_1),r(\tau_2)) = a(\tau_1) + \sum_{k=1}^{\infty} \frac{2(-1)^k}{\Pi k} \sin(\Pi k a(\tau_1)) \cos(\frac{2\Pi k \tau_2}{Tp}) \quad (9)$$

The pulsed signal $y(t)$ can be obtained by setting $\tau_1 = \tau_2 = t$ is given in equation 10

$$y(t)=a(t)+\sum_{k=1}^{\infty} \frac{2(-1)^k}{\Pi k} \sin(\Pi k a(t)) \cos(\frac{2\Pi k t}{Tp}) \quad (10)$$

Where $\Omega p = 2\Pi f_p = 2\Pi/Tp$ is the angular PWM frequency

III. ASYMMETRIC DOUBLE EDGE PWM

A digital pulse width modulator transforms the amplitude signal $a[n]$ which belongs to $[0,1]$ into a train of two level pulses $y[n]$ of different widths at a fixed pulse period Tp . Hence, the information lying in the amplitude of $a[n]$ is encoded in the widths of the pulses of $y[n]$ consists of asymmetric double edge PWM ,where the pulse train $y[n]$ consists of asymmetrical pulses that are centered around the midpoint of the pulse period Tp . The edges of the pulse can be determined by finding the intersection point of the amplitude signal $a[n]$ with a triangular reference wave $r[n]$ that is periodic in Tp . The amplitude signal $a[n]$ has to fulfill certain constraints. First the bandwidth of $a[n]$ must be adequately smaller than the reference wave frequency $f_p=1/Tp$. Second the amplitudes of $a[n]$ has to lie within the interval $[0,1]$. In order to use the described asymmetric double edge PWM for signals that do not meet the latter requirement, appropriate processing steps have to be applied. For a real valued signal proper scaling and adding bias could solve the problem. For a complex valued signal, a polar system architecture or an in-phase/quadrature component (IQ) system architecture can be implemented. In a polar architecture, PWM is performed on the magnitudes of the signal, and phase modulation is performed subsequently. In IQ architecture, PWM is performed on real imaginary parts of the signal separately, requiring a signal combined afterward.

The asymmetric double edge PWM operator $P(a[n], r[n])$ is given in equation 11.

$$P(a[n],r[n])=\{ 1,\text{for } a[n] \geq r[n] \text{ and } 0,\text{for } a[n] < r[n] \} \quad (11)$$

$$y[n]=a[n]+\sum_{k=1}^{\infty} \frac{2(-1)^k}{\Pi k} \sin(\Pi k a[n]) \cos(\frac{2\Pi k n}{Tp}) \quad (12)$$

Conventional digital asymmetric double edge PWM as per equation 3.12 can be implemented with low computational effort by using comparator. However, the non linear non band limited operation of the pulse width modulator inevitably induces aliasing in the resulting pulsed signal $y[n]$. The aliasing effect causes distortion in and around the frequency band of amplitude signal $a[n]$ from the pulsed signal $y[n]$, distortion remains within recovered signal. Due to aliasing effect, conventional PWM is not suitable for applications where the amplitude signal $a[n]$ has to be recovered with the high quality from the pulsed signal $y[n]$.Fig.1 represents asymmetric double edge PWM.

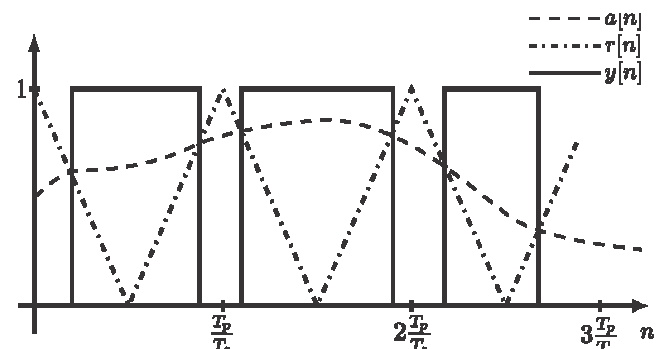


Fig.1. Asymmetric Double Edge PWM.

IV.AF- PWM

The AF-PWM method is based on conventional comparator PWM. To avoid the aliasing effect, the PWM operator $P(a[n],r[n])$ is slightly altered in a way that limit's the frequency content of the pulsed signal $y[n]$, which is given in equation 13.

$$y[n]=a[n]+\sum_{k=1}^K \frac{2(-1)^k}{\pi k} \sin(\pi k a[n]) \cos(\frac{2\pi k n}{T_p}) \quad (13)$$

Where K is the finite number that ensures the frequency content of the pulse width modulator output $y[n]$ is limited to an amount that avoid destructive aliasing in and around the frequency band of the aliasing signal $a[n]$.As a consequence the AF-PWM cannot be implemented by a comparator. Instead the pulsed signal $y[n]$ is obtained by direct implementation of the equation $y[n]$. In order to do so a large number of multiplications and sine and cosine computations are required ,where the computational effort increases with number of harmonics K . These operations might be costly when using fixed-point hardware such as application specific integrated circuits and field programmable gate arrays. Hence an implementation with low computational effort is required in order to make the AF-PWM a viable candidate for practical applications.

A. Implementation

In order to avoid multiplications and trigonometric functions, the use of the LUTs is targeted as an alternative.

$$y[n]= a[n]+ \sum_{k=1}^K \frac{(-1)^k}{\pi k} (\sin(2\pi k T_s n/T_p) + \pi k a[n]) - \sin(2\pi k T_s n/T_p) \pi k a[n] \quad (14)$$

By exploiting the 2π periodicity of sine waves the equation 14 can be split down into equation 15

$$y[n]=a[n]+y1[n]-y2[n] \quad (15)$$

$$y1[n]= \sum_{k=1}^K \frac{(-1)^k}{\pi k} (\sin(2\pi k T_s n/T_p) + \pi k a[n]) \quad (16)$$

$$y2[n]= \sum_{k=1}^K \frac{(-1)^k}{\pi k} (\sin(2\pi k T_s n/T_p) - \pi k a[n]) \quad (17)$$

$$\beta[n]=\text{mod}(T_s n/T_p,1) \quad (18)$$

Equations 16, 17and 18 are introduced to facilitate the convenient hardware realization. In order to implement the PWM with LUTs, the entries of the LUT has to be computed as per equation 19

$$ylut[I]= \sum_{k=1}^K \frac{(-1)^k}{\pi k} (\sin(2\pi k I/2^{N_{lut}})) \quad (19)$$

Where $2^{N_{lut}}$ is the number of values stored in the LUT and the index $I=0, \dots, 2^{N_{lut}} - 1$. Hence , a LUT comprises values for one complete saw tooth period. To obtain $y1[n]$ and $y2[n]$ from the LUTs, the indices $I1[n]$ and $I2[n]$ for the amplitude signal $a[n]$ at the current time index n need to be computed according to equations 20 and 21

$$I1[n]=[\text{mod}(\beta[n]+a[n]/2,1)2^{N_{lut}}] \quad (20)$$

$$I2[n]=[\text{mod}(\beta[n]-a[n]/2,1)2^{N_{lut}}] \quad (21)$$

Finally, the aliasing free pulse width modulator output can then be computed which can be mathematically expressed by equation 22

$$y[n]=a[n]+ylut[I1[n]]-ylut[I2[n]] \quad (22)$$

In hardware, the multiplications by $1/2^N$ and 2^N can be completely avoided by implementing them as arithmetic shift operations, denoted by $(x \gg N) = x/2^N$ and $(x \ll N) = x2^N$.The block diagram of Multiplier less Implementation of PWM is given in Fig.2

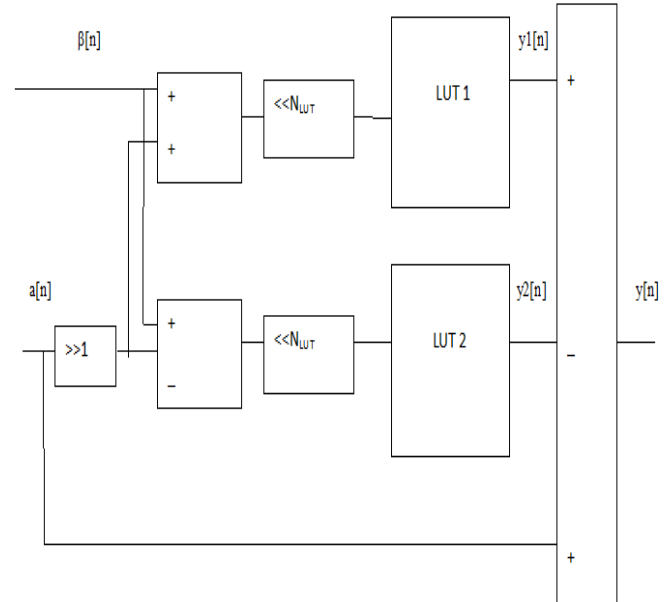


Fig.2. Block Diagram of AF-PWM (1-D).

V.PROPOSED BLOCK DIAGRAM

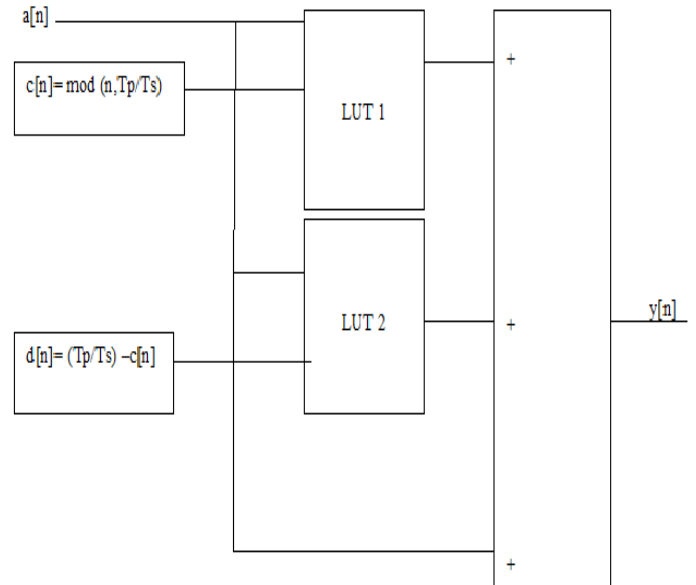


Fig.3. Block Diagram of AF-PWM (2-D).

Table I. Comparison of values of $y1[n]$ and $y2[n]$ for different $a[n]$ values.

	$a[n]= 0.1 \ \& \ k=10$		$a[n]= 0.2 \ \& \ k=10$	
$c[n]$	$y1[n]$	$y2[n]$	$y1[n]$	$y2[n]$
1	-0.1225	-0.0225	-0.1642	0.0348
2	-0.2007	-0.0862	-0.2455	-0.0408
3	-0.2630	-0.1630	-0.3256	-0.1209
4	-0.3227	-0.2444	-0.3810	-0.1882
5	-0.4092	-0.3093	-0.4108	-0.2483
6	-0.5669	-0.3616	0.5050	-0.3243
7	0.4388	-0.4388	0.4194	-0.4194
8	0.3616	0.5669	0.3243	-0.5050
9	0.3093	0.4092	0.2483	0.4108
10	0.2444	0.3227	0.1882	0.3810
11	0.1630	0.2630	0.1209	0.3256
12	0.0862	0.2007	0.0408	0.2455
13	0.0225	0.1225	-0.0348	0.1642
14	-0.0435	0.0435	-0.0979	0.0979

The table I shows the values of $y1[n]$ and $y2[n]$ are the same but with different phase shift, so in order to obtain $y2[n]$ from $y1[n]$ the indices $d[n]$ is used which performs the phase shift operation .

VI.SIMULATION RESULTS

The ratio of the parameters Tp and Ts is chosen as 14 for all the simulation results and the value of k is chosen differently. According to the simulation results it is evident that the values of $a[n]$ affects the width of pulse and the value of k is responsible for aliasing in and around the frequency band as per [8].

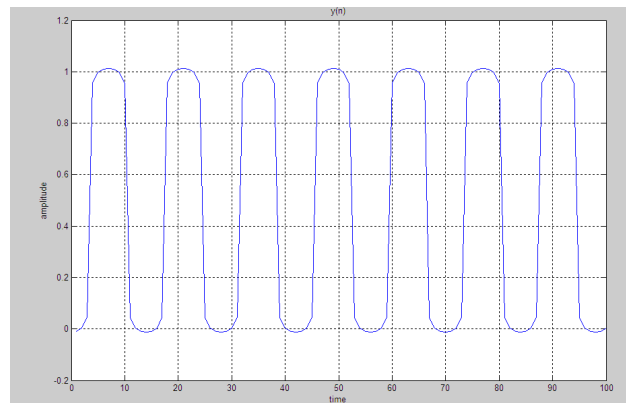


Figure 5. Simulation Result of Band Limited PWM of $y[n]$ for $a[n]=0.5$ and $K=25$.

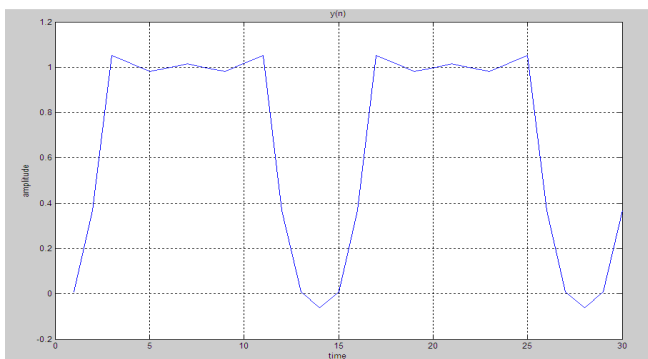


Figure 4. Simulation Result of Band Limited PWM of $y[n]$ for $a[n]=0.7$ and $K=10$.

The effect of amplitude ripples in the time domain pulsed signal can be minimized by intentionally choosing the number of components which is given in Figure 5 and 6.

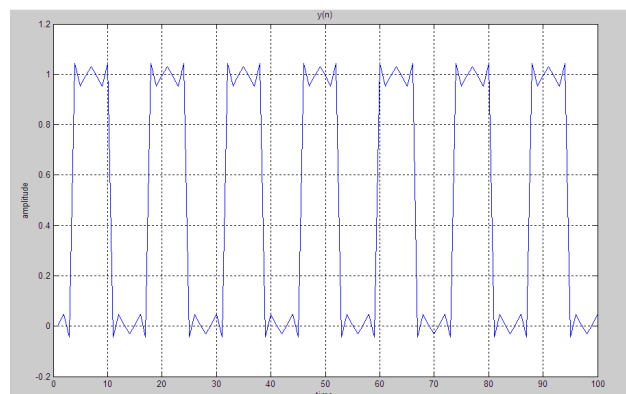


Figure 6. Simulation Result of Band Limited PWM of $y[n]$ for $a[n]=0.5$ and $K=10$.

VII.CONCLUSION

An alternative method of implementation of digital PWM that is based on the AF-PWM method [8] has been presented. For the implementation, LUTs and arithmetic shift operations are utilized in a way that allows for completely avoiding computationally expensive multiplications and trigonometric functions. Simulations demonstrate that the proposed implementation of the AF-PWM provides a simple technique with only low computational effort to implement high-DR digital PWM, which makes it suitable for applications employing switched PAs, such as digital audio or burst-mode RF transmitters. Two dimensional look up tables are used instead of one dimensional look up table in order to further reduce computational complexity.

VIII. REFERENCES

- [1] Adachi T., and Matsuura H., 2004, "A high efficiency transmitter with a delta-sigma modulator and a noise cancellation circuit," in Proc. 7th Eur. Conf. Wireless Technol., pp. 57–60.
- [2] Alarcon E., Garcia A., Guinjoan-Gispert F., Poveda A., and Tormo I., Oct. 2011, "Fundamental modulation limits for minimum switching frequency inband-error-free high-efficiency power amplifiers," IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 58, no. 10, pp. 2543–2555.
- [3] Allstot D.J., and Walling J.S., Feb. 2011, "Pulse-width modulated CMOS power amplifiers," IEEE Microw. Mag., vol. 12, no. 1, pp. 52–60.
- [4] Auger F., Feuvrie B., Li F., and Lou Z., Jul. 2011, "Multiplier-free divide, square root, log algorithms [DSP tips and tricks]," IEEE Signal Process. Mag., vol. 28, no. 4, pp. 122–126.
- [5] Belot D., Bercher J., Berland C., Hibon I., Le Goasoz V., Pache D., and Villegas M., Jan. 2006, "A transmitter architecture for nonconstant envelope modulation," IEEE Trans. Circuits Syst. II, Exp. Briefs, vol. 53, no. 1, pp. 13–17.
- [6] Berkhout M., and Dooper L., May 2010, "Class-D audio amplifiers in mobile applications," IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 57, no. 5, pp. 992–1002.
- [7] Blocher T., and Singerl P., Aug. 2009, "Coding efficiency for different switched-mode RF transmitter architectures," in Proc. 52nd IEEE Int. Midwest Symp. Circuits Syst., pp. 276–279.
- [8] Chi S., Hausmair K., Singerl P., and Vogel T., Feb. 2013 "Aliasing-free digital pulse-width modulation for burst-mode RF transmitters," IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 60, no. 2, pp. 415–427.
- [9] Eriksson T., Fager C., and Gustavsson U., Dec. 2010, "Quantization noise minimization in $\Sigma\Delta$ modulation based RF transmitter architectures," IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 57, no. 12, pp. 3082–3091.
- [10] Eriksson T., Fager C., Gustavsson U., Nemati H., Saad P., and Singerl P., Mar. 2012, "An RF carrier bursting system using partial quantization noise cancellation," IEEE Trans. Circuits Syst. I, Reg. Papers, vol. 59, no. 3, pp. 515–528.
- [11] Sarwate D.V., and Song Z., Oct. 2003, "The frequency spectrum of pulse width modulated signals," Signal Process., vol. 83, no. 10, pp. 2227–2258.
- [12] Chi S., Hausmair K., Singerl P., and Vogel T., Sep. 2013, "Multiplierless Implementation of an Aliasing Free Digital Pulse Width Modulator" IEEE Trans. Circuits Syst. II, vol.60, no.9, pp.592,596.