

Tsallis Entropy Based Multilevel Image Thresholding Using Chaotic Particle Swarm Optimization Algorithm

P.D. Sathya

Assistant Professor, Department of Electronics and Communication Engineering, Faculty of Engineering and Technology, Chidambaram – 608002, Tamilnadu, India

Abstract — The most fundamental step in image processing is image segmentation. This paper presents a Tsallis entropy based multilevel thresholding to find the desired content of an image of interest. Excellent result is obtained for bi-level thresholding and the exhaustive search for optimal threshold values for multilevel thresholding is reduced by objective function such as Tsallis based chaotic particle swarm optimization (Chaotic-PSO) metaheuristic algorithm drifting the pointer towards the fine-tuning search. The searching ability of the algorithm is improved by including the chaotic sequences of inertia weight factors of classical particle swarm optimization (PSO). The performance of Chaotic-PSO is analyzed extremely through comparison with other well-known global optimization techniques such as genetic algorithm (GA) and PSO. This proposed approach depicts its efficiency and effectiveness by testing on two axial, T2-weighted brain magnetic resonance image (MRI) test slices through quality measure indices such as Standard deviation, computational time, peak signal to noise ratio and uniformity measure to provide high quality segmented images.

Index Terms— Chaotic particle swarm optimization, magnetic resonance image, multi-level thresholding, Tsallis entropy.

I. INTRODUCTION

Image segmentation is the basic operation in the field of image processing to segment the meaningful part of an image based on its shape, size, color or texture [1]. Thresholding technique is most widely used segmentation method to predict the object of interest from its background. Thresholding finds its applications in Extraction of character from an image (or) to identify the defects of electronic components, to detect the tissue deformities causing abnormalities in brain region. Image with two classes can be split by bi-level thresholding whereas image with more than two classes will be partitioned by a simple, powerful, accurate and robust multilevel thresholding technique [2, 3]. This method utilizes the intensities for object identification. Pixels with same values of intensities describe the desired object of one class by multilevel thresholding. However, exponential increase in number of thresholds increases the computational time. Kapur based global thresholding has received considerable attention to work on image

segmentation. This problem is overcome by Tsallis based multilevel thresholding, capturing average information

content of events defining the entropy [4].

Advancement in information theory proposed various entropy models such as Shannon entropy, Renyi entropy, Tsallis's entropy and Cross entropy has been proposed for bilevel thresholding but, multilevel thresholding suffers from high computational cost resulting in exhaustive search for best threshold values. Nonconvexity and ruggedness reduces the convergence rate by Tsallis based multilevel thresholding. This Tsallis entropy calculates the cost function depending on total entropy criteria and maximizes it by global optimization algorithms. This Tsallis entropy maximizes the cross entropy between the standard and obtained segmented image. In this work, multilevel thresholding technique is introduced based on Tsallis aided with CPSO algorithm.

High quality solutions for difficult problems are provided by widespread metaheuristic algorithms [5-8]. PSO is a simple, easy and computational efficient algorithm, when compared with other heuristic techniques [9]. But PSO get stuck in local optima. To overcome this drawback, CPSO aided with Tsallis entropy is proposed in this paper. Global optimal point is reached by enhancing the ability of group interaction of particles through its ability to retain its past knowledge to return towards them when they deviate from the desired destination [10, 11].

II. OBJECTIVE FUNCTION OF TSALLIS ENTROPY

The Tsallis entropy [4] is based on the entropic thresholding technique of Kapur et al. due to the presence of non-additive information in some classes of images. The method substitutes a purely additive expression found in Kapur's original method for a pseudo-additive expression in the Tsallis theory.

A generalized form of Boltzmann-Gibbs-Shannon statistics proposed by Tsallis is expressed as:

$$S_q = \frac{1 - \sum_{i=1}^k p_i^q}{q - 1}$$

where k is the total number of possibilities of the system and the real number q denotes an entropic index that characterizes the degree of non-extensivity.

A. Bi-level thresholding

Tsallis bi-level thresholding can be described as follows:

$$J(t) = \text{argmax}[S_q^A(t) + S_q^B(t) + (1-q) \cdot S_q^A(t) \cdot S_q^B(t)] \quad (1)$$

where q is an entropic index, and

$$S_q^A(t) = \frac{1 - \sum_{i=0}^{t-1} \left(\frac{p_i}{p^A}\right)^q}{q-1}, \quad p^A = \sum_{i=0}^{t-1} p_i$$

$$S_q^B(t) = \frac{1 - \sum_{i=t}^{L-1} \left(\frac{p_i}{p^B}\right)^q}{q-1}, \quad p^B = \sum_{i=t}^{L-1} p_i$$

The information measures between the two classes (object and background) are maximized. When $S_q^A(t)$ is maximized, the luminance level t is considered to be the optimum threshold value and can be achieved by a cheap computational effort.

B. Multilevel thresholding

Tsallis entropy criterion method can also be extended to multilevel thresholding and is described as follows:

$$J(t) = \text{argmax}[S_q^A(t) + S_q^B(t) + S_q^C(t) + \dots + S_q^m(t) + (1-q) \cdot S_q^A(t) \cdot S_q^B(t) \cdot S_q^C(t) \cdot \dots \cdot S_q^m(t)] \quad (2)$$

where

$$S_q^A(t) = \frac{1 - \sum_{i=0}^{t_1-1} \left(\frac{p_i}{p^A}\right)^q}{q-1}, \quad p^A = \sum_{i=0}^{t_1-1} p_i$$

$$S_q^B(t) = \frac{1 - \sum_{i=t_1}^{t_2-1} \left(\frac{p_i}{p^B}\right)^q}{q-1}, \quad p^B = \sum_{i=t_1}^{t_2-1} p_i$$

$$S_q^C(t) = \frac{1 - \sum_{i=t_2}^{t_3-1} \left(\frac{p_i}{p^C}\right)^q}{q-1}, \quad p^C = \sum_{i=t_2}^{t_3-1} p_i \dots$$

$$S_q^m(t) = \frac{1 - \sum_{i=t_{m-1}}^{L-1} \left(\frac{p_i}{p^m}\right)^q}{q-1}, \quad p^m = \sum_{i=t_{m-1}}^{L-1} p_i$$

The CPSO based Tsallis entropy thresholding technique is used to determine the optimal threshold values. It can be performed through a search of the thresholds by optimizing the Tsallis objective function $J(t)$ using the CPSO algorithm.

III. CPSO ALGORITHM

A. Basic Concept of PSO Method

The most widely used particle swarm optimization algorithm is introduced by James Kennedy and Russel C. Eberhart (1995). The Intelligent metaheuristic algorithm finds its enormous applications in Machine learning, Data mining and in Image processing. PSO enhances the group interaction to reach the global optimum quickly and the 'memory' retention of previous knowledge trap the particles to return towards them [12].

In PSO, Particles move in D-dimensional space. Each particle ' i ' has a position $p_i = [p_{i,1}, p_{i,2}, \dots, p_{i,D}]$ and a velocity $v_i = [v_{i,1}, v_{i,2}, \dots, v_{i,D}]$.

In this, each particle keeps track of its co-ordinates in hyperspace to retain its fitness. The value of fitness stored is called ' $pbest$ ', Stored overall best value and its location are called ' $gbest$ ' and each particle keeps track of neighborhood called ' $lbest$ '.

The Standard PSO is described as:

$$V_{i,d}^{k+1} = W \times V_{i,d}^k + C_1 \text{rand}_1 \times (pbest_{i,d}^k - X_{i,d}^k) + C_2 \times \text{rand}_2 (gbest_d^k - X_{i,d}^k) \quad (3)$$

$$X_{i,d}^{k+1} = X_{i,d}^k + V_{i,d}^{k+1} \quad (4)$$

$$i = 1, 2, \dots, n; \quad d$$

where W is a weighting factor; C_1 is the acceleration factor; C_2 is a social acceleration factor; rand_1 and rand_2 are the random numbers between 0 and 1. $V_{i,d}^k$ is the velocity of particle i at iteration k ; $X_{i,d}^k$ is the d th dimension position of I at iteration k ; $pbest$ and $gbest$ are the d th dimension of own best and best particle at iteration k .

Time varying weighting function W is given by:

$$W = W_{max} - (W_{max} - W_{min}) \times \text{Iter} / \text{Iter}_{max} \quad (5)$$

where W_{max} and W_{min} are initial and final weight respectively, Iter is the current iteration number and Iter_{max} is the maximum iteration number.

B. CPSO

Chaotic behavior is depicted by equation:

$$f_k = \mu \cdot f_{k-1} \cdot (1 - f_{k-1}) \quad (6)$$

where μ is a control parameter with the range between $[0, 4]$.

Variation of μ makes an intense variation in behavior of the system. The above equation displays chaotic dynamics with $\mu=4$ and $f_0 \notin \{0, 0.25, 0.50, 0.75, 1.0\}$.

Here, global searching ability is improved by new weight:

$$W_{new} = W \times f \quad (7)$$

Weight parameter applied for chaotic sequences drives to reach the global optimal solution faster decreasing the computation time than PSO algorithm [13-14].

Step 1: The threshold value is taken as input.

Step 2: $W_{max}, W_{min}, C_1, C_2$ and Iter_{max} parameters are initialized.

Step 3: Initialize random positions and velocities for N particles

Step 4: Fitness value is calculated using the objective function (1) or (2)

Step 5: $pbest$ value is updated by comparing fitness value with $pbest$ and if the current value is better than $pbest$, then the current value is set as $pbest$.

Step 6: $gbest$ value is updated by comparing fitness value with $gbest$ and if the current value is better than $gbest$, then the current value is set as $gbest$.

Step 7: Chaotic weight W_{new}^{k+1} is updated

Step 8: Calculate the velocities V^{k+1} using equation (3)

Step 9: Calculate the velocities X^{k+1} using equation (4)

Step 10: Step 4 is continued until the current iteration reaches the maximum iteration number

Step 11: Last iteration output's the optimal solution

IV. PERFORMANCE EVALUATION

The effectiveness and robustness of the CPSO based Tsallis thresholding method is demonstrated on the axial, T2-

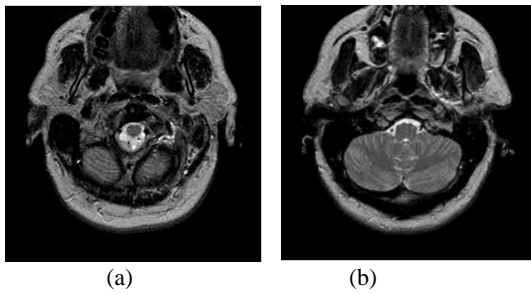


Fig. 1 T2 weighted brain MRI slices
(a) Slice 22, (b) Slice 32

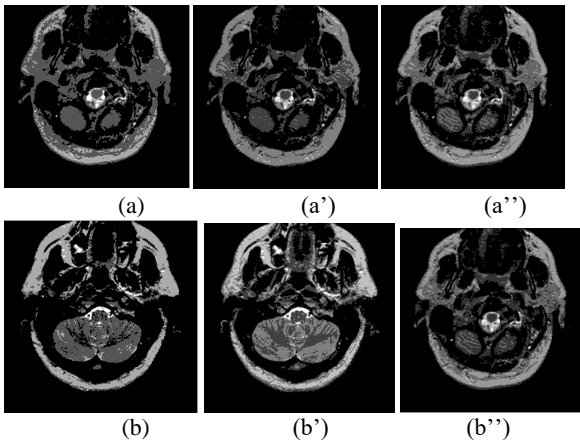


Fig. 2. Segmented brain MRIs with various threshold levels obtained by Tsallis Chaotic-PSO algorithm
[(a) and (b) represents 3-level thresholding, (a') and (b') represents 4-level thresholding, (a'') and (b'') represents 5- level thresholding]

Table 1 Comparison of objective values

Test images	m	Objective values		
		GA	PSO	CPSO
Brain MRI Slice#22	2	0.8649	0.8649	0.8649
	3	1.232429	1.243982	1.251429
	4	1.451651	1.453741	1.460244
	5	1.928470	1.933472	1.941798
Brain MRI Slice#32	2	0.8713	0.8713	0.8713
	3	1.232915	1.245165	1.253741
	4	1.543719	1.546813	1.555799
	5	1.934312	1.938360	1.941700

Table 2 Optimal threshold values

Test images	m	Optimal threshold values		
		GA	PSO	CPSO
Brain MRI Slice#2	2	94,176	94,176	94,176
	3	60,118,158	51,96,85	53,99,175
	4	59,95,134,179	65,107,146,193	69,112,149,193
	5	40,80,119,174,208	43,88,130,177,207	44,90,127,170,208
Brain MRI Slice#3	2	96,186	96,186	96,186
	3	84,148,203	100,144,194	89,128,186
	4	100,144,178,206	78,130,170,210	51,95,138,188
	5	80,111,161,191,203	76,113,134,159,200	68,106,136,184,218

Table 3 Comparison of standard deviation

Test images	m	Standard deviation value		
		GA	PSO	CPSO
Brain MRI Slice#22	2	7.7855 e-5	3.3637 e-6	1.1864e-6
	3	0.0153	0.0023	2.5244e-4
	4	0.0124	0.0052	5.8513e-4
	5	0.1142	0.0115	1.1286e-3
Brain MRI Slice#32	2	0.0000	0.0000	0.0000
	3	0.0127	0.0011	3.2671e-4
	4	0.0725	0.0076	8.8257e-4
	5	1.2543	0.0197	1.1569e-3

Table 4 Comparison of computation time (in seconds)

Test images	m	Computation time		
		GA	PSO	CPSO
Brain MRI Slice#22	2	4.2031	4.1406	3.7344
	3	4.7969	4.5626	3.8563
	4	5.5607	5.1980	4.1664
	5	5.6746	5.2835	4.4034
Brain MRI Slice#32	2	4.0574	3.9991	3.5658
	3	4.5127	4.3210	3.9817
	4	4.8024	4.5897	4.0206
	5	5.2651	4.8979	4.3885

Table 5 Comparison of PSNR values

Test images	m	PSNR values (db)		
		GA	PSO	CPSO
Brain MRI Slice#22	2	10.1212	10.1212	10.1212
	3	12.1883	12.5987	13.0012
	4	13.8215	14.0101	14.1383
	5	15.3025	16.2683	16.4021
Brain MRI Slice#32	2	10.5362	10.5362	10.5362
	3	14.7435	14.9672	15.0184
	4	16.0157	16.2018	16.7521
	5	16.2385	16.8366	17.1110

Table 6 Comparison of uniformity measure

Test Images	m	Uniformity measure		
		GA	PSO	CPSO
Brain MRI Slice#22	3	0.9482	0.9538	0.9604
	5	0.9219	0.9675	0.9711
Brain MRI Slice#32	3	0.9511	0.9547	0.9584
	5	0.9623	0.9685	0.9731

weighted brain MRI slices together with comparisons on GA and PSO algorithms. The test images are presented in Figure 1.

The objective values and the corresponding optimal threshold values obtained by GA, PSO, and Chaotic-PSO algorithms are tabulated in Tables 1 and 2 respectively. The simulation results reveal that MBF algorithm can provide better solutions than the other algorithms. Fig. 2 shows the CPSO based Tsallis's thresholding of brain MRI for 3, 4 and 5 threshold levels. It is obvious that the segmentation quality is improved for higher threshold levels.

The Table 3 summarizes the standard deviation obtained by the GA, PSO and Chaotic-PSO for fifty runs. It can be seen that the standard deviation obtained by CPSO algorithm is least as compared to other algorithms, emphasizing the stability of the algorithm.

The Table 4 reveals the computation time of the various algorithms. The simulation results in Table 4 illustrate that the computation time of the CPSO algorithm is significantly small with respect to GA and PSO and offers a general idea that the CPSO based Tsallis thresholding method can be utilized in practical applications.

The PSNR value is high for five level thresholding of the brain MRI images using CPSO algorithm as indicated from the PSNR values acquired by GA, PSO, and Chaotic-PSO algorithms detailed in Table 5.

The uniformity measures obtained by the GA, PSO, and CPSO algorithms in Table 6 are compared for the brain MRIs. It is evident that the results obtained by the CPSO algorithm for five level thresholding are better for both the images than those using GA, and PSO algorithms, the reason being that their objective functions are higher.

V. CONCLUSION

The CPSO algorithm has been applied for solving multilevel thresholding problem, with an endeavor to maximize the Tsallis entropy function. The utility of the proposed algorithm has been demonstrated by considering brain MRIs images, and compared with other evolutionary algorithms such as GA and PSO algorithms. The simulation results have been found to confirm the potential of the CPSO algorithm in solving multilevel thresholding problem and portray its effectiveness for practical applications. The quantitative results such as PSNR and uniformity measure of two different kinds of images confirm that the proposed CPSO algorithm offers superior image quality measurement values compared to its GA and original PSO algorithm. Future work is to apply the CPSO algorithm to different image segmentation methods and extend it to many other aspects of image processing.

REFERENCES

- [1] N. R. Pal. S. K. Pal, "A review on image segmentation techniques", *Pattern Recognition*, Vol. 26, year 1993, pp 1277-1294,
- [2] J. N. Kapur, P. K. Sahoo, A. K. C. Wong, "A new method for gray-level picture thresholding using the entropy of the histogram", *Computer Vision Graphics and Image Processing*, Vol. 29, year 1985, pp 273-285.
- [3] N. Otsu, "A threshold selection method from gray-level histograms", *IEEE Transactions on Systems, Man, Cybernetics SMC-9*, year 1979, pp 62-66.
- [4] P.D. Sathya, "Optimum Multilevel Image Thresholding Based on Tsallis Entropy Method with Bacterial Foraging Algorithm", *International Journal of Computer Science Issues*, Vol. 7, Issue 5, September 2010, pp. 336-343.
- [5] P-Y. Yin, "A fast scheme for optimal thresholding using genetic algorithms", *Signal Processing*, Vol. 72, year 1999, pp 85-95.
- [6] L.K. Huang, M. J. Wang, "Image thresholding by minimizing the measure of fuzziness", *Pattern Recognition*, Vol. 28, year 1995, pp 41-51.
- [7] [8] H. D. Cheng, J. Li, "Threshold selection based on fuzzy c-partition entropy approach", *Pattern Recognition*, Vol. 31, year1998, pp 857-870.
- [8] J. Kittler, J. Illingworth, "Minimum error thresholding", *Pattern Recognition*, Vol. 19, year 1986, pp 41-47.
- [9] R.C. Eberhart and J. Kennedy, "Particle swarm optimization", *IEEE Int. Con. Neural Networks*, vol. 4, pp 1942-1947, year 1995.
- [10] Y. Shi and R.C. Eberhart, "A modified particle swarm optimizer," *IEEE Int. Con. Evolutionary Computations*, pp. 69-73, year1998.
- [11] Y. Shi and R.C. Eberhart, "Empirical study of particle swarm optimization," *IEEE Int. Proc. Evolutionary Computations*, vol. 3, pp. 1945-1946, year 1999.
- [12] Z.L. Gaing, "Particle swarm optimization to solving the economic dispatch considering the generator constraints," *IEEE Trans. Power Systems*, vol. 18, no. 1, pp. 1187-1195, year 2003.
- [13] Liu Bo, Wang Ling, Jing Ti-Hui, Tang Fung, and Huang De-Xian, "Improved particle swarm optimization combined with chaos solutions," *Chaos Solutions and Fractals*, pp. 1261-1271, year 2005.
- [14] Liu Bo, Wang Ling, Jing Ti-Hui, Tang Fung, and Huang De-Xian, "Improved particle swarm optimization combined with chaos solutions", *Chaos Solutions and Fractals*, pp. 1261-1271, year 2005.