

Restrained Triple Connected Domination Number of Cartesian Products of Paths

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Abstract:

In [5] Paulraj Joseph J., Angel Jebitha M.K., Chithra Devi P. and Sudhana G.introduced the concept of Triple connected graphs. In [4], G. Mahadevan, A. Selvam, V. G. Bhagavathi Ammal and T. Subramanian, introduced the concept of restrained triple connected domination number of a graph with real life applications in computer communication networks. Moreover very new interesting real life applications in computer communication network can be developed by imposing some conditions. In this paper we commence the Restrained Triple Connected Domination Number of the Cartesian product of path graphs. The restrained dominating set is said to be restrained triple connected dominating set, if $\langle S \rangle$ is triple connected. The minimum cardinality taken over all the restrained triple connected dominating sets is called the restrained triple connected domination number and is denoted by $\gamma_{rtc}(G)$. We determine the restrained triple connected domination numbers of $P_m \square P_n$.

Key words: Restrained Triple connected domination number of a graph, Cartesian product of path.

AMS Subject Classification: 05C

1 .Introduction

By a *graph* we mean a finite, simple, connected and undirected graph $G(V, E)$. A sub set S of V of a nontrivial graph G is called a *dominating set* of G if every vertex in $V - S$ is adjacent to at least one vertex in S . A graph G is said to be triple connected if any three vertices lie on a path in G . A dominating set is said to be restrained dominating set if every vertex in $V - S$ is adjacent to at least one vertex in S as well as another vertex in $V - S$. The restrained dominating set is said to be restrained triple connected dominating set, if the $\langle S \rangle$ is triple connected. The minimum cardinality taken over all the restrained triple connected dominating sets is called the restrained triple connected domination number and is denoted by $\gamma_{rtc}(G)$.

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Definition 1.1 The Cartesian product of paths P_m and P_n with disjoint set of vertices V_m and V_n and Edge sets E_m and E_n is the graph with vertex set $V(P_m \square P_n)$ and edge set $E(P_m \square P_n)$ such that any two vertices (u_1, u_2) and (v_1, v_2) are adjacent in $G \square H$ if and only if either (i) $u_1 = v_1$ and u_2 is adjacent to v_2 in H . (ii) $u_2 = v_2$ and u_1 is adjacent to v_1 in G , where $G \square H$ denotes the Cartesian product of graphs G and H .

In this paper we afford the restrained triple connected domination number of Cartesian product of path. For a fixed i , the set $(P_m)_i = P_m \diamond i$ is called a column of $P_m \square P_n$ (i^{th} column of $P_{m,n}$), the set $j(P_n) = j \diamond P_n$ is called a row of $P_m \square P_n$ (j^{th} row of $P_{m,n}$). $(i, j)P_m$ denotes the row by column format.

Definition 1.2 Let C_1 and C_2 be two cycles of four vertices. Suppose C_1 has vertex set $\{x_1, x_2, x_3, x_4\}$ and C_2 has vertex set $\{y_1, y_2, y_3, y_4\}$ then H-merging of C_1 and C_2 having the vertex $\{x_1, x_2 = y_1, x_3, x_4 = y_3, y_2, y_4\}$ edge set including all the edges C_1 and C_2 and $(x_2, x_4) = (y_1, y_3)$.

Definition 1.3 V-merging of C_1 and C_2 having the vertex $\{x_1, x_2, x_3 = y_1, x_4 = y_2, y_3, y_4\}$ edge set including all the edges C_1 and C_2 and $(x_3, x_4) = (y_1, y_2)$. From H-merging and V-merging, $G_{2,3} = G_{2,2} \square G_{2,2}$ and $G_{3,2} = G_{2,2} \square G_{2,2}$

H-merging of two grid graphs:

Let $G_{2,4}$ have vertex set $\{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and $G_{2,3}$ have vertex set $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ then the vertex set $G_{2,4} \square G_{2,3}$ is $\{u_1, u_2, u_3, u_4 = v_1, v_2, v_3, u_5, u_6, u_7, u_8 = v_4, v_5, v_6\}$ and edge set includes all the edges

and $(u_4, u_8) = (v_1, v_4)$. That is, $G_{2,4} \square G_{2,3} = G_{2,6}$ Generally, $G_{2,(k-1)} \square G_{2,2} = G_{2,(k-1+2-1)} = G_{2,k}$. Also $G_{2,k} \square G_{2,k} = G_{3,k}$

Observation 1.3 Let S_1 and S_2 be the RTCD sets $P_{m,s}$ and $P_{m,t}$ respectively. Then $\gamma_{rtc}(P_{m,s+t}) \leq s + t$

Observation 1.4 The RTCD set of complete grid graphs are find out when $m \geq 2$ and $n \geq 3$. To find the RTCD number for $G_{2,3}$, we have to merge $G_{2,2}$ and $G_{2,2}$.

Observation 1.5 It's obvious to find the $\gamma_{rtc}(G_{2,n})$ by the repeated application of $G_{2,2}$. For the complete grid graph $P_m \square P_n$, if $m = 1$, then it's a path graph P_n . The RTCD number of a path graph is n

2. Main Result

Theorem 2.1 The RTCD number of a grid graph $P_2 \square P_n$ for $n \geq 3$ is $\gamma_{rtc}(P_2 \square P_n) = n$.

Proof: consider the grid graph $G_{2,n} = P_2 \square P_n$ for $n \geq 3$. For $n = 3$, the grid graph $G_{2,3}$ is a domino graph of RTCD number 3. Hence, $\gamma_{rtc}(P_2 \square P_3) = 3$. Suppose the result is true for $n = k - 1$. That is, $\gamma_{rtc}(P_2 \square P_{k-1}) = k - 1$. To prove the result is true for $n = k$. Consider the grid graph $G_{2,k-1} = P_2 \square P_{k-1}$, and then one vertex from each pair $(u_1, v_1), (u_2, v_2), (u_3, v_3) \dots \dots \dots, (u_{k-1}, v_{k-1})$ will be in the RTCD set. Hence $\gamma_{rtc}(P_2 \square P_{k-1}) = k - 1$.

We have $G_{2,k} = G_{2,2} \square G_{2,k-1}$. Thus one pair of vertex is added to $G_{2,k-1}$ and one vertex of the pair should be added the RTCD set. Hence $\gamma_{rtc}(G_{2,k}) = k - 1 + 1 = k$.

$$\gamma_{rtc}(G_{2,n}) = n$$

Alternatively, by the definition of RTCD set, each vertex of one pair will be in $\gamma_{rtc}(G)$, without loss of generality, let it be $\{u_1, u_2, u_3, \dots, u_n\}$. Hence, $\gamma_{rtc}(G_{2,n}) \leq n$. By taking alternative vertices of the pairs, $\gamma_{rtc}(G)$ includes all the vertices excluding (u_1, v_1) and (u_2, v_2) .

$$\gamma_{rtc}(G_{2,n}) = 2n - 4 \geq n$$

$$\text{Hence } \gamma_{rtc}(G_{2,n}) = n.$$

For instance,

$$\gamma_{rtc}(G_{2,7}) = \gamma_{rtc}(G_{2,4} \square G_{2,4}) = 4 + 4 - 1 = 7$$

.

Continuing like this,

$$\gamma_{rtc}(G_{2,n}) = \gamma_{rtc}(G_{2,n-2} \square G_{2,3}) = n - 2 + 3 - 1 = n.$$

The RTCD set of $G_{3,n}$ can be obtained by V-merging of two cycles such as D_1 and D_4 to form T_1 and T_4 which is minimal RTCD set of $G_{3,n}$. T_1 can be obtained by the repeated merging of D_1 and for the general case; V-merging can be classified as,

$$G_1: T_1 \square T_1 \square \dots \dots T_1;$$

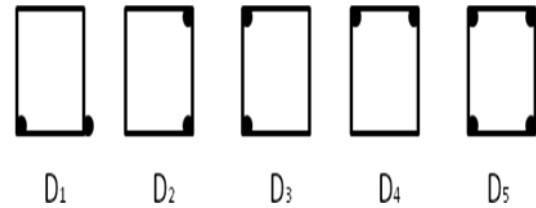
$$G_2: T_2 \square T_2 \square \dots \dots T_2;$$

$$G_3: T_3 \square T_3 \square \dots \dots T_3;$$

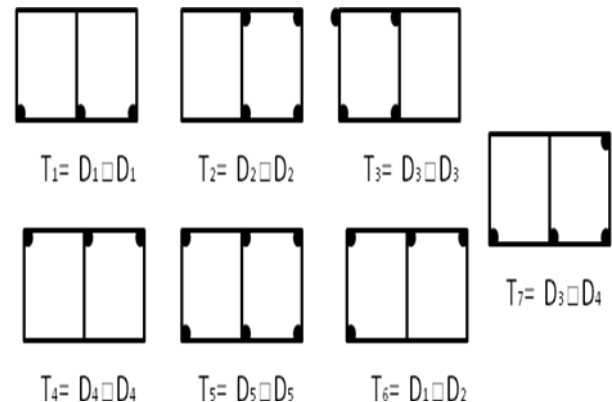
Corollary 2.2 The RTCD number of a grid graph $P_3 \square P_n$ for $n \geq 3$ is

$$\gamma_{rtc}(G_{3,n}) = \gamma_{rtc}(P_3 \square P_n) = 2n.$$

Proof: Consider the grid graph $G_{2,n} \square G_{2,n} = G_{3,n}$ for $n \geq 3$. Consider the possible ways of the dominating sets of cycles are,



Then the possible ways for RTCD set for the complete grid graph are,



$$G_4: T_4 \square T_4 \square \dots \dots T_4;$$

$$G_5: T_5 \square T_5 \square \dots \dots T_5;$$

$$G_6: T_6 \square T_6 \square \dots \dots T_6;$$

$$G_7: T_7 \square T_7 \square \dots \dots T_7;$$

$$G_8: T_2 \square T_2 \square \dots \dots T_3;$$

$$G_9: T_6 \square T_6 \square \dots \dots T_7;$$

The cardinalities of above mentioned merging graphs are,

$$\begin{aligned}
 |G_1| : |T_1 \square T_1 \square \dots \square T_1| &= n \\
 |G_2| : |T_2 \square T_2 \dots \square T_2| &= 2n - 2; \\
 |G_3| : |T_3 \square T_3 \dots \square T_3| &= 2n - 2; \\
 |G_4| : |T_4 \square T_4 \dots \square T_4| &= n; \\
 |G_5| : |T_5 \square T_5 \dots \square T_5| &= 2n; \\
 |G_6| : |T_6 \square T_6 \dots \square T_6| &= 2n - 2; \\
 |G_7| : |T_7 \square T_7 \dots \square T_7| &= 2n - 2; \\
 |G_8| : |T_2 \square T_2 \dots \square T_3| &= 2n - 4; \\
 |G_9| : |T_6 \square T_6 \dots \square T_7| &= 2n - 4;
 \end{aligned}$$

Hence,

From $G_1, G_2, G_3, G_4, G_6, G_7, G_8,$
 $G_9, \gamma_{rtc}(G_{3,n}) = \gamma_{rtc}(P_3 \square P_n) \leq 2n.$

From
 $G_5, \gamma_{rtc}(G_{3,n}) = \gamma_{rtc}(P_3 \square P_n) \geq 2n.$

Thus
 $\gamma_{rtc}(G_{3,n}) = \gamma_{rtc}(P_3 \square P_n) = 2n$

Corollary 2.3 The RTCD number of grid graph $P_4 \square P_n$ for $n \geq 4$

is
 $\gamma_{rtc}(G_{4,n}) = \gamma_{rtc}(P_4 \square P_n) = 2n + 2$

Proof: Consider the grid graph $G_{2,n} \square G_{3,n} = G_{4,n}$ for $n \geq 2$. That is, the RTCD set of $G_{4,n}$ can be obtained by V-merging of

minimal dominating set of $G_{2,n}$ and $G_{3,n}$ gives the minimal RTCD set of $G_{4,n}$. Hence
 $\gamma_{rtc}(G_{4,n}) = \gamma_{rtc}(G_{2,n}) + \gamma_{rtc}(G_{3,n}) = [2n]$

Alternatively, the possible ways of taking dominating sets are
 $[(1,1), (1,2) \dots (1,n) \cup (4,1), (4,2) \dots (4,n) (P_n) \cup (2,n) \cup (3,n)]$
 OR $[2(P_n) \text{ and } 3(P_n)]$ OR
 $[2(P_n) \text{ and } 4(P_n)].$

If
 $[(1,1), (1,2) \dots (1,n) \cup (4,1), (4,2) \dots (4,n) (P_n) \cup (2,n) \cup (3,n)]$
 is a dominating set,

then $\gamma_{rtc}(P_4 \square P_n) = 2n + 2$. That is, $\gamma_{rtc}(P_4 \square P_n) \leq 2n + 2$. If $[2(P_n) \text{ and } 3(P_n)]$ OR $[2(P_n) \text{ and } 4(P_n)]$ as a dominating set then RTCD number of the above cases are $3n, 3n$.

$$\gamma_{rtc}(P_4 \square P_n) \leq 2n + 2 \text{ then } \gamma_{rtc}(G_{4,n}) = \gamma_{rtc}(P_4 \square P_n) = 2n + 2$$

Corollary 2.4

$$\gamma_{rtc}(G_{5,n}) = \gamma_{rtc}(P_5 \square P_n) = 2n + 2, n \geq 5.$$

Proof: consider the grid graph $P_5 \square P_n$, each row dominates the two neighboring rows. Thus $\lceil \frac{m}{3} \rceil$ rows dominate all the rows. For satisfying RTCD set, in between vertices of $(P_m)_1$ and $(P_m)_n$ will also be in RTCD set. That is, $\lceil n + 2 \rceil$. Thus

$$\gamma_{rtc}(P_5 \square P_n) = n \left\lceil \frac{m}{3} \right\rceil + [n + 2] = \left\lceil \frac{nm}{3} + n + 2 \right\rceil.$$

For the complete grid graph $P_5 \square P_n$, $2(P_n)$ and $4(P_n)$, then $\gamma_{rtc}(P_5 \square P_n) \leq 2n + 2$

If $1(P_n)$ and $4(P_n)$ or $2(P_n)$ and $5(P_n)$ as RTCD then

$$\gamma_{rtc}(P_5 \square P_n) = 2n + 4 \geq 2n + 2$$

Hence

$$\gamma_{rtc}(G_{5,n}) = \gamma_{rtc}(P_5 \square P_n) = 2n + 2$$

Generalizing this result for even values of m , i.e., $m=2k$, for all even values of m , each

Complete Grid Graph	Merging pattern	RTCD number
$G_{2,n}, n \geq 3$	$G_{2,n}$	n
$G_{3,n}$	$G_{2,n} \square G_{2,n}$	$2n$
$G_{4,n}$	$G_{2,n} \square G_{3,n}$	$2n + 2$
$G_{5,n}$	$G_{3,n} \square G_{3,n}$	$2n + 2$
$G_{6,n}$	$G_{3,n} \square G_{2,n} \square G_{2,n}$	$2n + 2$
$G_{7,n}$	$G_{4,n} \square G_{4,n}$	$3n + 4$

Theorem 2.5 The RTCD number of a grid graph $G_{m,n} = P_m \square P_n$ for $n \geq 7, k \geq 3$ is,

row $i(P_m)$ dominates two neighbouring rows. Thus, the RTCD number includes $\left\lceil \frac{m}{3} \right\rceil$,

and the merging pattern of $G_{2k,n} = G_{k,n} \square G_{k,n} \square \dots \square G_{k-1,n} \Rightarrow$

$$\gamma_{rtc}(G_{2k,n}) = n \text{ times of } (k - 1) + \left\lceil \frac{m}{3} \right\rceil = n(k - 1) + \left\lceil \frac{2k}{3} \right\rceil$$

for all odd values of m , that is, $m=2k+1$, by the same

way

$$G_{2k,n} = G_{k,n} \square G_{k,n} \square \dots \square G_{k,n} \Rightarrow \gamma_{rtc}(G_{2k,n}) = n \text{ times of } k + \left\lceil \frac{m}{3} \right\rceil = nk + \left\lceil \frac{2k+1}{3} \right\rceil$$

By combining all the above results, the merging pattern and RTCD number are tabulated as follows.

$G_{8,n}$	$G_{4,n} \square G_{4,n}$	$3n + 6$
$G_{5,n}$		
$G_{2k,n}$	$G_{k,n} \square G_{k,n} \square \dots \square G_{k,n}$	$n(k - 1)$
$G_{2k+1,n}$	$G_{k,n} \square G_{k,n} \square \dots \square G_{k,n}$	$n(k - 1)$

$$\gamma_{rtc}(P_m \square P_n) = \begin{cases} \left\lfloor \frac{m(n+2)}{3} \right\rfloor, m = 3k \\ \left\lfloor \frac{(m+2)(n+2)}{3} \right\rfloor, m = 3k + 1 \\ \left\lfloor \frac{(m+1)(n+2)}{3} \right\rfloor, m = 3k + 2 \end{cases}$$

$$\gamma_{rtc}(P_n \square P_n) = \begin{cases} \left\lfloor \frac{n(n+2)}{3} \right\rfloor, m = 3k \\ \left\lfloor \frac{(n+2)^2}{3} \right\rfloor, m = 3k + 1 \\ \left\lfloor \frac{(n+1)(n+2)}{3} \right\rfloor, m = 3k + 2 \end{cases}$$

Proof:

Case 1: If $m = 3k, k \geq 3$. $G_{3k,n}$ Can be formed by V- merging of $G_{k,n} \square G_{k,n} \square \dots \square G_{k+1,n}$. Each $i(P_n)$ dominates its two neighboring rows. For satisfying the triple connected domination the in between vertices of the first and last column will also be in RTCD set.

$$\gamma_{rtc}(G_{3k,n}) = k \text{ times of } (n + 2) = k(n + 2) = \left\lfloor \frac{m}{3} (n + 2) \right\rfloor, m = 3k$$

Case 2: If $m = 3k + 1, k \geq 3$, means one row added to the grid graph. Thus the RTCD set is k times of $(n+2)+(n+2)$.

$$\gamma_{rtc}(G_{3k+1,n}) = k \text{ times of } (n + 2) + (n + 2) = (n + 1)(k + 1) = (n + 2) \left\lfloor \frac{(m-1)}{3} + 1 \right\rfloor = \left\lfloor \frac{(m+2)(n+2)}{3} \right\rfloor$$

Case 3: If $m = 3k + 2, k \geq 3$, means two rows added to the grid graph. Thus,

$$\gamma_{rtc}(G_{3k+2,n}) = k \text{ times of } (n + 2) + (n + 2) = (n + 1)(k + 1) = (n + 2) \left\lfloor \frac{(m-2)}{3} + 1 \right\rfloor = \left\lfloor \frac{(m+1)(n+2)}{3} \right\rfloor$$

Corollary 2.6 The RTCD number of a $n \times n$ square grid graph $G_{n,n} = P_n \square P_n$ for $k \geq 3$, is,

Observation 2.7 $\gamma_{rtc}(P_m) \times \gamma_{rtc}(P_n) = mn$, and for all the above cases $\gamma_{rtc}(P_m \square P_n) \leq \gamma_{rtc}(P_m) \times \gamma_{rtc}(P_n)$, Vizing conjecture does not satisfied for the Cartesian product of two paths.

Conclusion: In this paper we provided the restrained triple connected domination number of Cartesian products of path graphs. The authors obtained similar results for various different types of products in the literature, which will be reported in the subsequent papers.

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