International Journal of Emerging Technology in Computer Science & Electronics (IJETCSE) ISSN: 0976-1353 Volume 26 Issue 4 – JUNE 2019.

Secure Degree Equitable Dominating Graph

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Abstract:
Let G = (V, E) be a graph. Let v ∈ V: The open neighbourhood N(v) and closed neighbourhood N[v] are defined by N(v) =
{u∈ V : uv ∈ E} and N[v] = N(v) ∪ { v}

A set contains D is the secure degree equitable dominating set in G it satisfies the following conditions

i) A vertex u∈ V is set to be degree equitable with a vertex v∈ V if | deg(u) – deg(v)|≤ 1.
ii) A dominating set S of G is a secure dominating set if for each u ∈ V - S there exists v ∈ N(u) ∩ S such that (S –{v}) ∪ {u} is a secure dominating set.

The minimum cardinality of the secure degree equitable dominating set D_{sde}(G) and is denoted by δ_s^{de}(G). This paper find the secure degree equitable domination number of δ_s^{de}(G) of cycle graphs, path graphs and complete graphs and also find the

Keywords: *Dominating graph, Secure Domination, Equitable dominating,*

I. Introduction:

Let G = (V, E) be a graph, mean a finite undirected graph with neither loops nor multiple edges.

Let G = (V, E) be a graph, it is said to be complete all the vertices of V are adjacent to each other.

Degree of a vertex v ∈V can be defined number of edges incident with v, it can be denoted by deg(v).

Let G = (V, E) be a graph. Let v ∈ V: The open neighbourhoodN(v) and closed neighbourhood N[v] are defined by N(v) =
{u∈ V : uv ∈ E} and N[v] = N(v) ∪ { v}

II. Theorem and Proof:

The set S is called a dominating set of G if every vertex in V - S is adjacent to at least one vertex in S.
A dominating set S of
G is a secure dominating set if for each u ∈ V - S there exists
v∈ N(u) ∩ S such that (S –{v}) ∪ {u} is a dominating set.

A vertex u∈ V is set to be degree equitable with a vertex v∈ V if

| deg(u) – deg(v)|≤ 1.

A subset *D* of *V* is called an equitable dominating set if for every*v* ∈ *V* - *D* there exists *a* vertex *u* ∈ *D* such that
uv ∈ *E*(*G*) and
| deg(u) – deg(v)|≤ 1.

A set contains D is the secure degree equitable dominating set in G it satisfies the following conditions

iii) A vertex u∈ V is set to be degree equitable with a vertex v∈ V if |deg(u) – deg(v)|≤ 1.
iv) A dominating set S of G is a secure dominating set if for each u ∈ V - S there exists v ∈ N(u) ∩ S such that (S –{v}) ∪ {u} is a secure dominating set.

The minimum cardinality of the secure degree equitable dominating set is called secure degree equitable domination number and is denoted byδ_s^{de}.

Theorem 1: Let $K_n$be a complete graph of order m≥2, then δ_s^{de} (K_m) =1.

Proof: Given a secure degree equitable doming set D_{sde} of K_m= {x_1, x_2, x_3, ….,x_m}, assume that S = {x_1} ∈ D_{sde}, Then x_1 dominates all other vertices in K_m, and since K_m is a complete graph, then for every $x_{i\neq 1}$ ∈ K_m, x_1 and $x_{i\neq 1}$ are adjacent and

|deg(x_1) – deg($x_{i\neq 1}$)|= 0Hence

$|\deg(x_1) - \deg(x_{i\neq 1})| \leq 1$

For each

$x_{i\neq 1} \in V - S = \{x_2, x_3, \ldots\ldots, x_m\}$ there exists $x_{i\neq 1} \in N(x_1) \cap S$ such that

$(S - \{x_{i\neq 1}\}) \cup \{x_1\}$ is a secure dominating set.

Then D_{sde} (K_m) = { x_1} which gives δ_s^{de} (K_m) =1.

For example

K_4 is a complete graph (in Figure 1) with V ={a, b, c, d}.

Let S= {a} and V- S={b, c, d} and

| deg (a) - {deg(b)or deg(c) or deg(d)}|=0. Therefore K_4 is degree equitable.

For each x (say d) belongs to V- S = {b, c, d} and N (d) = {a, c, b} and N (b) ∩ S = {d} and then (S-{a}) ∪ {b} = {b, c, d} is a secure dominating set. δ_s^{de} (K_4) =1.

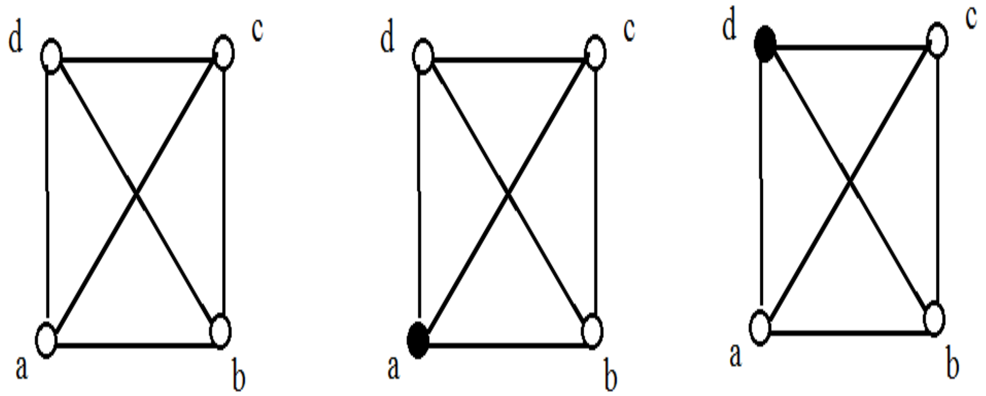

Figure 1

K_4 (Removing a vertex {a} from S and inserting the adjacent vertex d to S. Therefore K_4 is secure degree equitable domination.δ_s^{de} (K_4) =1.)

Theorem 2: Let $P_n$be a path graph of order n (n ≥2), then δ_s^{de} (P_n) is greatest integer function of($\frac{n}{2}$).

Proof:

Every vertex in P_n degree is either 2(between vertices) or 1(endvertex). Therefore clearly P_n is degree equitable.

For n=2,

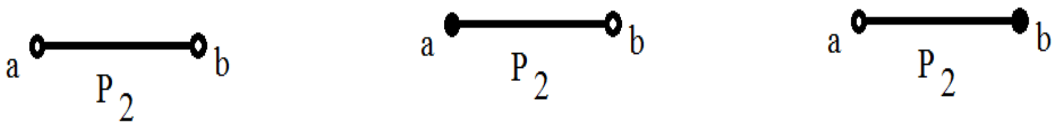

Figure 2

In P_2 (In Figure 2), S = {a}.Removing a vertex {a} from S and inserting the adjacent vertex {b}to S. Then S = {b} is a secure dominating set. Therefore δ_s^{de} (P_2) is greatest integer function of $\left(\frac{2}{2}\right)$= 1.

For n = 3,

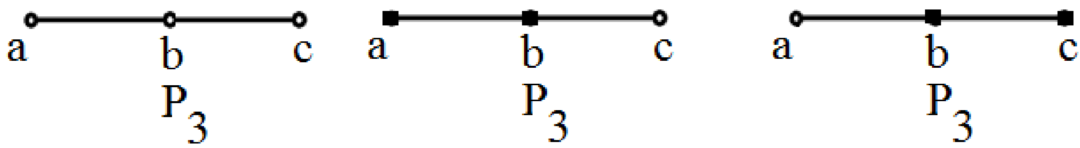

Figure 3

In P_3(in Figure 3), S = {a, b}. Removing a vertex {b} from S and inserting the adjacent vertex {c} to S. Then S={c, b} is a secure dominating set. Therefore δ_s^{de} (P_3) is greatest integer function of $\left(\frac{3}{2}\right)$= 2.

For n= 4,

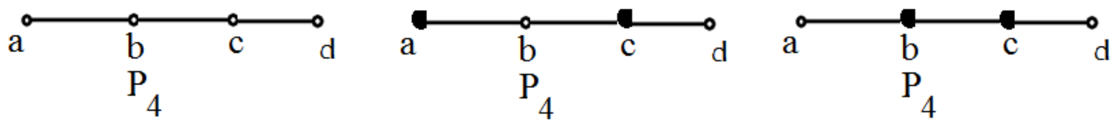

Figure 4

In P_4 (Figure 4), S = {a, c}. Removing a vertex {a} from S and inserting the adjacent vertex {b} to S. Then S={c, b} is a secure dominating set.

Therefore δ_s^{de} (P_4) is greatest integer function of $\left(\frac{4}{2}\right)$= 2.

For n = 5,

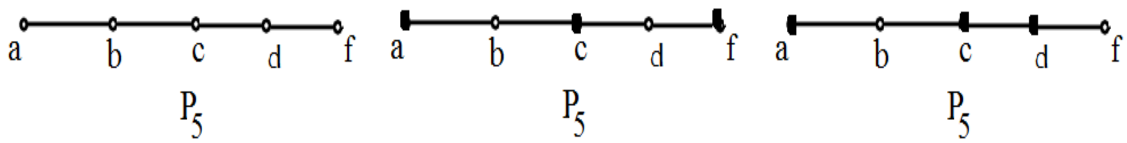

Figure 5

In P_5 (in Figure 5), S = {a, c, f}. Removing a vertex {f} from S and inserting the adjacent vertex {d} to S. Then S={a, c, d} is a secure dominating set.Therefore δ_s^{de} (P_5) is greatest integer function of $\left(\frac{5}{2}\right)$= 3.

For n = 6,

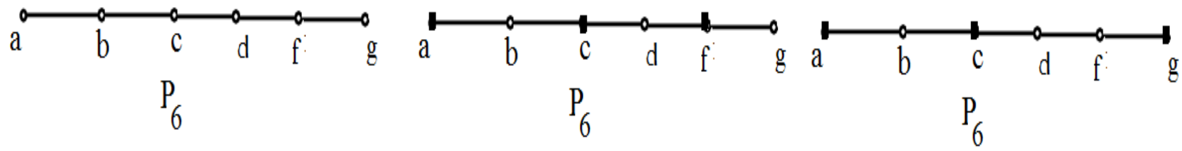

Figure 6

In P_6, (in Figure 6) S = {a, c, f}. Removing a vertex {f} from S and inserting the adjacent vertex {g} to S. Then S={a, c, g} is a secure dominating set.Therefore δ_s^{de} (P_6) is greatest integer function of $\left(\frac{6}{2}\right)$= 3.

Theorem 3: Let $C_n$be a complete graph of order n≥3, then δ_s^{de} (C_n) is greatest integer function of$(\frac{n}{2})$.

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Therefore in general P_n be a path graph of order n, then δ_s^{de} (P_n) is greatest integer function of$(\frac{n}{2})$.

Proof:

Every vertex in C_n is degree of 2. Therefore clearly C_n is degree equitable.

For n = 3,

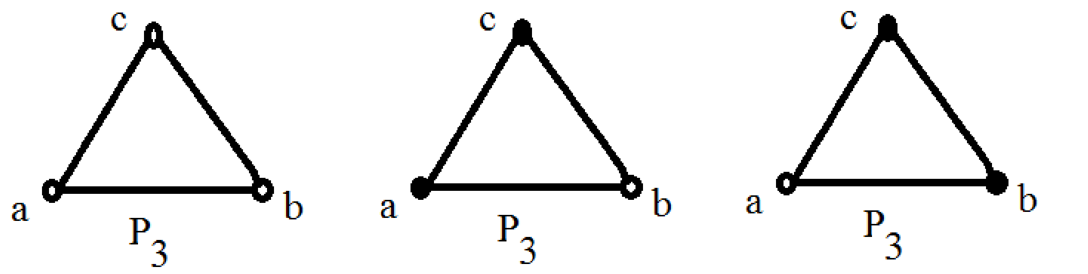

Figure 7

In C_3 (In Figure 7)
, S = {a, c}. Removing a vertex {a} from S and inserting the adjacent vertex {b} to S. Then S={c, b} is a secure dominating set. Therefore δ_s^{de} (C_3) is greatest integer function of $\left(\frac{3}{2}\right)$= 2.

For n = 4,

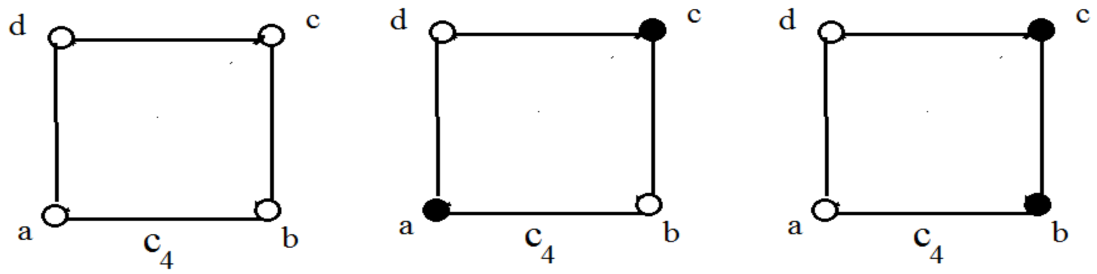

Figure 8

In C_4 (In Figure 8)
, S = {a, c}. Removing a vertex {a} from S and inserting the adjacent vertex {b} to S. Then S={c, b} is a secure dominating set. Therefore δ_s^{de} (C_4) is greatest integer function of $\left(\frac{4}{2}\right)$= 2.

For n = 5,

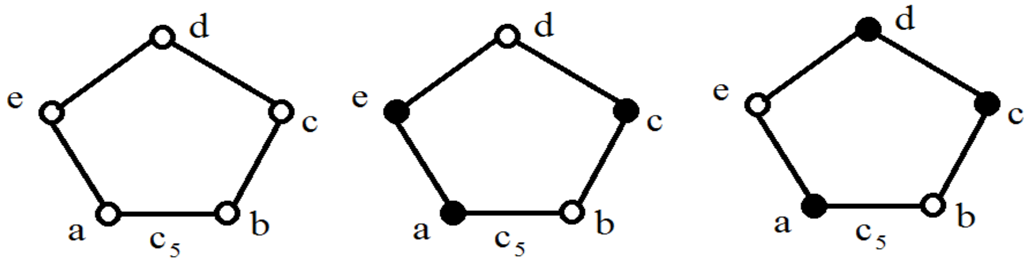

Figure 9

In P_5 (In Figure 9)
, S = {a, c, e}. Removing a vertex {e} from S and inserting the adjacent vertex {d} to S. Then S={a, c, d} is a secure dominating set. Therefore δ_s^{de} (P_5) is greatest integer function of $\left(\frac{5}{2}\right)$= 3.

For n = 6,

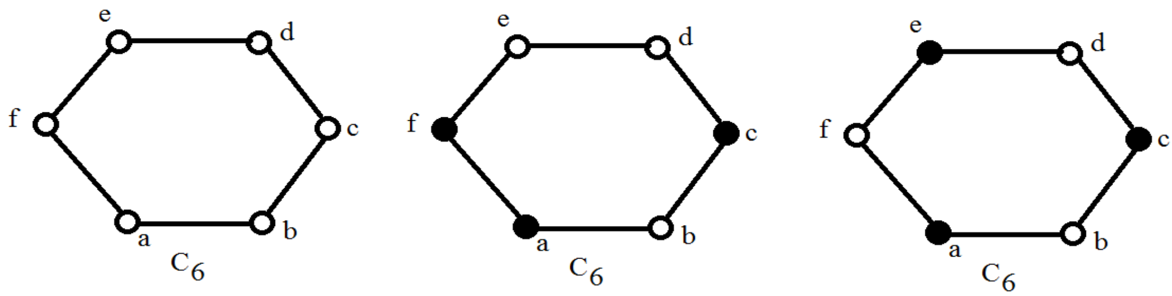

Figure 10

In C_6 (In Figure 10)
, S = {a, c, f}. Removing a vertex {f} from S and inserting the adjacent vertex {e} to S. Then S={a, c, e} is a secure dominating set. Therefore δ_s^{de} (C_6) is greatest integer function of $\left(\frac{6}{2}\right)$= 3.

III. Conclusion:

From this paper we can able to find the δ_s^{de}(G) of cycle graphs, path graphs and complete graphs and also find the secure degree equitable domination number.

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